Duke Math Meet 2016

Relay Round 1

1. If x, y are two distinct real numbers and the two sequences (x, a_1, a_2, a_3, y) and $(b_1, x, b_2, b_3, y, b_4)$ are both arithmetic sequences. Suppose

$$\frac{b_4 - b_3}{a_2 - a_1} = \frac{p}{q}$$

where $\frac{p}{q}$ is in lowest terms. Calculate $p + q$.

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2. Let $T = TNYWR$. How many distinct real solutions are there for the equation

$$(x^2 - 7x + T)^{(x^2 - 1)} = 1?$$

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3. David and $TNYWR$ other people are passing a ball around. Every second, the ball is passed at random to another person. Suppose David starts off with the ball. After five seconds, what is the probability that David has the ball?

1. Given that

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}.$$

Find the value of

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2}$$

.

2. Let $T = \frac{\pi^2}{TNYWR}$. How many ways can we split T distinguishable toys among Justin, David, and Trung? A toy may be shared by two children but not by all three children.

3. Let $TNYWR = 6^T$ and $n = 2T + 5$. There are n people $a_1, .., a_n$ in a room and they shake hands with each other such that for all $1 \le i \le n - 1$, a_i shook hands i times. How many times did person a_n shake hands?