Duke Math Meet 2016
Individual Round

1. Trung took five tests this semester. For his first three tests, his average was 60, and for the fourth test he earned a 50. What must he have earned on his fifth test if his final average for all five tests was exactly 60?

2. Find the number of pairs of integers (a, b) such that $20a + 16b = 2016 - ab$.

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3. Let $f : \mathbb{N} \to \mathbb{N}$ be a strictly increasing function with $f(1) = 2016$ and $f(2t) = f(t) + t$ for all $t \in \mathbb{N}$. Find $f(2016)$.

4. Circles of radius $7, 7, 18$, and r are mutually externally tangent, where $r = \frac{m}{n}$ for relatively prime positive integers m and n. Find $m + n$.

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5. A point is chosen at random from within the circumcircle of a triangle with angles $45^o, 75^o, 60^o$. What is the probability that the point is closer to the vertex with an angle of 45^o than either of the two other vertices?

6. Find the largest positive integer a less than 100 such that for some positive integer b, $a - b$ is a prime number and ab is a perfect square.

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7. There is a set of 6 parallel lines and another set of six parallel lines, where these two sets of lines are not parallel with each other. If Blythe adds 6 more lines, not necessarily parallel with each other, find the maximum number of triangles that could be made.

8. Triangle ABC has sides $AB = 5$, $AC = 4$, and $BC = 3$. Let O be any arbitrary point inside ABC, and $D \in BC$, $E \in AC$, $F \in AB$, such that $OD \perp BC$, $OE \perp AC$, $OF \perp AB$. Find the minimum value of $OD^2 + OE^2 + OF^2$.

9. Find the root with the largest real part to $x^4 - 3x^3 + 3x + 1 = 0$ over the complex numbers.

10. Tony has a board with 2 rows and 4 columns. Tony will use 8 numbers from 1 to 8 to fill in this board, each number in exactly one entry. Let array $(a_1, .., a_4)$ be the first row of the board and array $(b_1, .., b_4)$ be the second row of the board. Let $F = \sum_{i=1}^{4} |a_i - b_i|$, calculate the average value of F across all possible ways to fill in.